Spin Liquid and Superconductivity Emerging from Steady States and Measurements

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We demonstrate that, starting with a simple fermion wave function, the steady mixed state of the evolution of a class of Lindbladians, and the ensemble created by strong local measurement of fermion density without postselection can be mapped to the “Gutzwiller projected” wave functions in the doubled Hilbert space—the representation of the density matrix through the Choi-Jamiołkowski isomorphism. A Gutzwiller projection is a broadly used approach of constructing spin liquid states. For example, if one starts with a gapless free Dirac fermion pure quantum state, the constructed mixed state corresponds to an algebraic spin liquid in the doubled Hilbert space. We also predict that for some initial fermion wave function, the mixed state created following the procedure described above is expected to have a spontaneous “strong-to-weak” U(1) symmetry breaking, which corresponds to the emergence of superconductivity in the doubled Hilbert space. We also design the experimental protocol to construct the desired physics of mixed states.

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Introduction—Quantum spin liquids, a class of highly nontrivial disordered phase of quantum spins, have been the subject of an extremely active subfield of condensed matter physics since their early theoretical proposal [1,2]. Despite great progress made in the field, many open questions and challenges remain (for a review, please refer to [3–5]). First of all, the theoretical description of spin liquids usually involves a strongly coupled gauge theory, which is a formidable analytical problem except for certain theoretical limits. Second, though it is certain that spin liquids do exist in some elegant theoretical models, e.g., the Kitaev model [6], numerical simulation of quantum spin liquid on more realistic models often suffers from sign problems as the models that potentially realize the spin liquid usually have geometric frustration. Hence, controversies continue to persist about the existence or nature of the spin liquids in various important realistic models. Third, the signal of quantum spin liquid in real correlated materials may be obscured by the inevitable disorders and impurities.

Rather than the parent Hamiltonian, one can instead focus on the spin liquid wave function. One standard construction of spin liquid wave function is the so-called Gutzwiller projected state [7–13]:

$$|\mathrm{SL}\rangle = \prod_i \hat{\mathcal{P}}(\hat{n}_{i,\uparrow} + \hat{n}_{i,\downarrow} = 1)|f_{i,\uparrow}, f_{i,\downarrow}\rangle, \tag{1}$$

where $|f_{i,\uparrow}, f_{i,\downarrow}\rangle$ is a simple spin-1/2 fermion state, and it can be often taken as a free fermion state with on average one fermion per site. The projection $\hat{\mathcal{P}}$ ensures that there is one and only one fermion per site, which matches the onsite Hilbert space of a spin-1/2 system.

Though the Gutzwiller projection is broadly used as a trial mean field wave functions of spin liquids, it is never exactly realized in condensed matter systems. In this Letter we demonstrate that the Gutzwiller wave function can be realized in a completely different setup: it can be constructed as the steady mixed state of a Lindbladian evolution, or as the ensemble created by strong measurements of local operators, starting with a simple fermion wave function. The mixed state density matrix obtained from both constructions in the doubled Hilbert space (the Choi-Jamiołkowski representation [14,15]), becomes exactly a Gutzwiller wave function. Predictions of the constructed mixed state will be made based on our theoretical understanding of quantum spin liquids.

We also demonstrate that sometimes the constructed mixed state density matrix is expected to become a supercondutor in the doubled Hilbert space, which corresponds to the “strong-to-weak” spontaneous symmetry breaking, a subject that has attracted great interests very recently [16–20]. These theoretical predictions and expectations can be tested using experimental protocols designed in Supplemental Material (SM) [21].

Basic formalism—Construction with Lindbladian: We consider the nonunitary Lindbladian evolution of a density matrix. Let us assume that the density matrix at $t = 0$ corresponds to a pure quantum state $\rho_0 = |\Psi_0\rangle\langle\Psi_0|$ of fermions. For simplicity we will start with a noninteracting spinless fermion wave function $|\Psi_0\rangle$, and we assume that $|\Psi_0\rangle$ is the ground state of a Hamiltonian H_0 on a d-dim lattice: $H_0 = \sum_{\langle i,j\rangle} -t_{ij} c_i^\dagger c_j + \mathrm{H.c.}$ We will first assume that H_0 is a *gapless* free fermion tight-binding model, later we will discuss interactions in H_0.

We consider the situation where the jump operators are the local fermion density operators $L_i = \sqrt{\gamma}\hat{n}_i = \sqrt{\gamma} c_i^\dagger c_i$. The nonunitary evolution of a density matrix under a general Lindbladian reads (we assume that the unitary part of the evolution is absent)

$$\partial_t \rho(t) = \sum_i L_i \rho L_i^\dagger - \frac{1}{2}\left(L_i^\dagger L_i \rho + \rho L_i^\dagger L_i\right). \tag{2}$$

Exploring physics of mixed states using the Choi-Jamiołkowski isomorphism of the density matrix, i.e., representing the density matrix as a pure quantum state in the doubled Hilbert space [14,15], has attracted a great deal of interest [22–30]. The Choi-Jamiołkowski isomorphism maps a density matrix $\rho = \sum_n p_n |\psi_n\rangle\langle\psi_n|$ to a state $|\rho\rangle\rangle \sim \sum_n p_n |\psi_{n,L}\rangle|\psi_{n,R}\rangle$. In our current case it is also convenient to take the Choi-Jamiołkowski representation of the density matrix. The evolution of the Choi state in the doubled Hilbert space is given by

$$|\rho_t\rangle\rangle \sim e^{t\mathcal{L}}|\rho_0\rangle\rangle, \tag{3}$$

where

$$\begin{aligned}\mathcal{L} &= \sum_i \gamma\left(\hat{n}_{i,L}\hat{n}_{i,R} - \frac{1}{2}\left(\hat{n}_{i,L}^2 + \hat{n}_{i,R}^2\right)\right) \\ &= \sum_i -\frac{\gamma}{2}\left(\hat{n}_{i,L} - \hat{n}_{i,R}\right)^2. \end{aligned}\tag{4}$$

Here, L and R label two copies of fermionic modes in the doubled Fock space of the Choi-Jamiołkowski isomorphism of the density matrix. The initial Choi state $|\rho_0\rangle\rangle$ for the pure state $\rho_0 = |\Psi_0\rangle\langle\Psi_0|$ is the free fermion state with the parent Hamiltonian

$$\begin{aligned}\mathcal{H}_0 &= H_0(c_{i,L}) + H_0^*(c_{i,R}) \\ &= \sum_{\langle i,j\rangle} -t_{ij} c_{i,L}^\dagger c_{j,L} - t_{ij}^* c_{i,R}^\dagger c_{j,R} + \mathrm{H.c.}\end{aligned}\tag{5}$$

In the limit $t \to \infty$, the steady state $|\rho_\infty\rangle\rangle$ satisfies the constraint $\hat{n}_{i,L} - \hat{n}_{i,R} = 0$.

To more explicitly reveal the implication of the constraint $\hat{n}_{i,L} - \hat{n}_{i,R} = 0$, let us perform a particle-hole (PH) transformation on $c_{i,R}$, and formally relabel the fermions in the L, R spaces as "↑" and "↓":

$$\begin{aligned} c_{i,L} &\to f_{i,\uparrow}, \qquad c_{i,R} \to \eta_i f_{i,\downarrow}^\dagger, \\ \hat{n}_{i,L} &\to \hat{n}_{i,\uparrow}, \qquad \hat{n}_{i,R} \to 1 - \hat{n}_{i,\downarrow}, \end{aligned}\tag{6}$$

where $\eta_i = \pm 1$ can be chosen to depend on the sites i. Then in the limit $t \to \infty$, the constraint $\hat{n}_{i,L} - \hat{n}_{i,R} = 0$ becomes

$$\hat{n}_{i,\uparrow} + \hat{n}_{i,\downarrow} = 1, \tag{7}$$

which is precisely the Gutzwiller projection. In this case, the steady Choi state is mapped to a spin wave function in the standard spin liquid literature:

$$|\rho_\infty\rangle\rangle = \prod_i \hat{\mathcal{P}}(\hat{n}_{i,\uparrow} + \hat{n}_{i,\downarrow} = 1)|\rho_0\rangle\rangle, \tag{8}$$

where $\hat{\mathcal{P}}(\hat{n}_{i,\uparrow} + \hat{n}_{i,\downarrow} = 1)$ is the projector onto the subspace with single occupation. Since now on every site there is one and only one fermion $f_{i,\alpha}$, the projected wave function $|\rho_\infty\rangle\rangle$ lives in an effective spin-1/2 Hilbert space, with spin-1/2 operators $\hat{S}_i^\mu = \frac{1}{2} f_{i,\alpha}^\dagger \sigma_{\alpha\beta}^\mu f_{i,\beta}$. Under certain conditions, for example, (1) the fermions in $|\Psi_0\rangle$ are at half-filling, i.e., there are on average 1/2 fermions per site; and (2) under particle-hole transformation H_0^* becomes H_0 with a certain choice of η_i, then the parent tight-binding Hamiltonian of $|\rho_0\rangle\rangle$ in the doubled Hilbert space enjoys a full effective SU(2) "spin" symmetry. For example, if H_0 is a tight-binding model on a bipartite lattice with nearest neighbor hopping, then choosing $\eta_i = -1$ on one sublattice will result in a SU(2) symmetric $\mathcal{H}_0$ in Eq. (5).

Construction with measurements: When the jump operators are Hermitian and all commute with each other, the steady state density matrix also describes the ensemble of wave functions generated by strong measurements on the jump operators *without postselection*. For example, when we perform strong measurement of local densities $\hat{n}_i$ on a pure fermion quantum state $|\Psi_0\rangle$, *without* postselecting the measurement outcomes, the density matrix of the ensemble becomes

$$\rho = \sum_{\mathbf{n}} \hat{P}_{\mathbf{n}} \rho_0 \hat{P}_{\mathbf{n}}, \tag{9}$$

where $\mathbf{n}$ represents a configuration of fermion numbers $\hat{n}_i$, and $\hat{P}_{\mathbf{n}}$ is a projection operator to the particular configuration. Such an ensemble can be obtained in Fermi gas microscope with high spatial resolution [31–37].

Through measurements, the system wave function collapses to the eigenstate of local density. Without postselection, the final mixed state is a classical mixture of eigenstates of local density with all eigenvalues, which becomes a quantum superposition in the doubled space, except now the fermion density in the left and right spaces are identified. This classical mixture of density eigenstates is precisely given by the Gutzwiller projection in the doubled space.

We also note that construction of spin liquids as the doubled state of a mixed state was discussed before [28–30], but to the best of our knowledge, the general connection to the Gutzwiller wave function, which is a broadly used construction of spin liquids, has not been explored.

Dirac spin liquid—A spin liquid state typically involves an emergent dynamical gauge field [7–13,38]. When the emergent gauge symmetry is U(1), and the fermions $f_{i,\alpha}$ have Dirac nodes at the Fermi level, the low energy action describing the spin liquid is a QED$_3$ with Dirac fermion matter fields coupled to a U(1) gauge field [39].

QED$_3$ Dirac spin liquids have attracted great interest [8,9,12,38,41–45]. The dynamical gauge field A_μ may lead to confinement and hence destabilize the spin liquid states. In our construction, to ensure a deconfined phase of the gauge field, one can start with a state $|\Psi_0\rangle$ that is the ground state of N flavors of degenerate fermions $c_{i,I}$ at half-filling, where $I = 1 \cdots N$. For example, one can start with a state of the alkaline earth cold atoms, which enjoy a large flavor symmetry [46]. Then eventually the effective Gutzwiller projection imposed on the mixed state in the doubled space is

$$\sum_{I=1\cdots N} \hat{n}_{i,I,\uparrow} + \hat{n}_{i,I,\downarrow} = N, \tag{10}$$

which ensures that $|\rho\rangle\rangle$ is a wave function of an effective SU$(2N)$ spin system, with self-conjugate representation of SU$(2N)$ on each site.

Assuming the dispersion of $|\Psi_0\rangle$ has Dirac nodes in the momentum space with N-fold flavor degeneracy, the spin liquid that $|\rho\rangle\rangle$ simulates is described by the following theory:

$$\mathcal{S} = \int d^2x d\tau \sum_{v=1}^{N_v} \sum_{a=1}^{2N} \bar{\psi}_{a,v} \gamma_\mu (\partial_\mu - \mathrm{i} A_\mu) \psi_{a,v} + \frac{1}{4e^2} F_{\mu\nu}^2. \tag{11}$$

Here, v labels the N_v Dirac nodes and valleys in the momentum space. For example, if $|\Psi_0\rangle$ is the ground state of a nearest neighbor tight binding model on the honeycomb lattice at half-filling, or on the square lattice with π flux per square, then $N_v = 2$, i.e., there are two independent Dirac cones in the momentum space. It is known that for large enough N, Eq. (11) describes a stable algebraic liquid state which is also a $(2+1)d$ conformal field theory. We refer to this liquid state in the doubled Hilbert space as the algebraic Choi-spin liquid.

Predictions for the Choi-spin liquid described above can be made based on our understanding of the spin liquid. Let us still start with a SU$(2N)$ spin liquid on the square lattice with π flux, whose low energy physics is described by QED$_3$ in Eq. (11). The local spin operator $\hat{S}_i^z = \sum_{I=1\cdots N} \hat{n}_{i,I,\uparrow} - \hat{n}_{i,I,\downarrow}$ has a nonzero overlap with the fermion-bilinear composite field $\bar{\psi} S^z \mu^z \psi$, where $\mu^z = \pm 1$ denotes the two Dirac valleys. $\bar{\psi} S^z \mu^z \psi$ is an SU$(2N)$ Néel order parameter. Then at long distance the spin correlation functions is given by

$$\langle \hat{S}_0^z \hat{S}_{\boldsymbol{x}}^z \rangle \sim (-1)^{\boldsymbol{x}} \frac{1}{|\boldsymbol{x}|^{2\Delta}} + \cdots. \tag{12}$$

Note that the sign of the correlation function above oscillates with the sublattice of $\boldsymbol{x}$. With sufficiently large N, the scaling dimension Δ can be computed using the standard $1/N$ expansion, and it is smaller than the scaling dimension of the free Dirac fermion [47]:

$$\Delta = 2 - \frac{32}{3NN_v\pi^2} + O\left(\frac{1}{N^2}\right). \tag{13}$$

We have conducted a numerical study on the Gutzwiller projected π-flux state on the square lattice, and we see a power-law scaling with a considerably smaller scaling dimension compared with free Dirac fermion (Fig. 1), qualitatively consistent with the prediction of QED$_3$, and consistent with previous numerics [48]. If we take $N_v = 2$ in Eq. (13), the first order $1/N$ expansion leads to power-law $2\Delta_1 \sim 2.92$ and $2\Delta_2 \sim 3.46$ for $N = 1, 2$ respectively, both feature enhanced correlation compared with the free Dirac fermion (with $2\Delta = 4$). And $N = 2$ gives better agreement with the numerics (Fig. 1), as expected.

The SU$(2N)$ spin correlation function above can be used to make predictions for the following "Renyi-2" correlation in the current context:

$$\begin{aligned} \langle\langle \rho | \delta\hat{n}_{0,\uparrow} \delta\hat{n}_{\boldsymbol{x},\uparrow} | \rho \rangle\rangle &\sim \langle\langle \rho | \hat{S}_0^z \hat{S}_{\boldsymbol{x}}^z | \rho \rangle\rangle \\ &\sim \mathrm{tr}(\rho^2 \delta\hat{n}_0 \delta\hat{n}_{\boldsymbol{x}}) \sim (-1)^{\boldsymbol{x}} \frac{1}{|\boldsymbol{x}|^{2\Delta}}. \end{aligned} \tag{14}$$

Here, $\delta\hat{n}(\boldsymbol{x}) = \hat{n}(\boldsymbol{x}) - N/2$, and we have used the fact that $\delta n_\uparrow(\boldsymbol{x}) \sim \hat{S}_{\boldsymbol{x}}^z/2$, since $\hat{n}_\uparrow(\boldsymbol{x}) + \hat{n}_\downarrow(\boldsymbol{x})$ is a constant. Since the

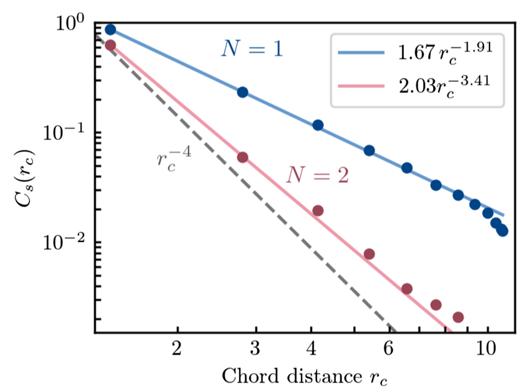

FIG. 1. The staggered spin-spin correlation of the projected π-flux state on the square lattice (24×24 torus), which is a Dirac spin liquid constructed with $N = 1$ and $N = 2$ species of fermions. The power-law exponent is considerably enhanced compared with the unprojected wave function (dashed line), consistent with predictions of spin liquid theory. The analytical calculation based on $1/N$ expansion is given in Eq. (13); note that there is another "valley number" N_v in the equation. In our case $N_v = 2$.

Choi state $|\rho\rangle\rangle$ has a SU($2N$) symmetry, all the SU($2N$) spin correlation function should have the same scaling as Eq. (14). For example,

$$\mathrm{tr}\Big(\rho^2(\hat{O}_0)^I_J(\hat{O}_{\boldsymbol{x}})^J_I\Big) \tag{15}$$

is expected to decay in the same manner as Eq. (14) for sufficiently large N, where $\hat{O}^I_J = c^\dagger_I c_J$ with $I \neq J$ is a SU(N) operator that operates on the index $I = 1 \cdots N$. Note that $\hat{O}$ does not change the total number configuration $\mathbf{n}$.

Experimental protocol—The correlation Eq. (14) can be probed experimentally through the protocol detailed in SM [21], which we briefly summarize here. Our goal is to verify whether the prepared experimental system corresponds to a spin liquid in the doubled space. Building on insights from QED$_3$, the nature of the actual experimental system can be verified by comparing the "classical-classical" (CC) and the "quantum-classical" (QC) correlator proposed in Ref. [49]. Compared with other scenarios, the advantage of our proposal is that we do not require a precise simulation of the actual interacting experimental system in the classical computer, which is often intractable. In our case, it suffices to simulate a noninteracting reference system. Below is a summary of the key steps in our proposed protocol, and a detailed discussion is provided in SM [21]: (1) Prepare state $|\Psi_0\rangle$ (fermionic atoms in optical lattice) with parent Hamiltonian H_0. H_0 has a gapless spectrum, and can include short-range interactions. (2) In each experimental run, measure the fermion density on each site, obtain density configuration $\mathbf{n}_m$. (3) Input $\mathbf{n}_m$ in the computer, find $p_{\mathbf{n}_m} = \langle\tilde{\Psi}_0|\hat{P}_{\mathbf{n}_m}|\tilde{\Psi}_0\rangle$, where $|\tilde{\Psi}_0\rangle$ is the *free fermion* state whose parent Hamiltonian is the noninteracting part of H_0. (4) Average $p_{\mathbf{n}_m}\delta n_{0,m}\delta n_{\boldsymbol{x},m}$ over all experimental runs, and the result corresponds to the QC correlator. (5) Independently compute the CC correlator using $|\tilde{\Psi}_0\rangle$. If the CC correlator and QC correlator exhibit the same power-law scaling, based on our understanding of QED$_3$, the experimentally prepared $|\rho\rangle\rangle$ should be a spin liquid state, and its scaling is captured by the QC and CC correlators.

SU(2) gauge symmetry and superconductivity—It is well known in the field of spin liquid that (see, for example, Ref. [50]), a U(1) projection may actually lead to SU(2) gauge symmetry. In our context, for a class of free fermion states $|\Psi_0\rangle$ with $N = 1$ whose parent Hamiltonian H_0 only has real hopping t_{ij}, the seemingly U(1) gauge projection in the doubled space $\prod_i \hat{\mathcal{P}}(\hat{n}_{i,\uparrow} + \hat{n}_{i,\downarrow} = 1)$ would also lead to a SU(2) gauge symmetry. For example, the π-flux state with $N = 1$ in Fig. 1 actually has SU(2) gauge symmetry.

To expose the explicit SU(2) gauge symmetry, we no longer need the PH transformation of $H^*_0(c_{i,R})$. Instead, we just need to (trivially) relabel $L \to 1$, $R \to 2$, then the U(1) gauge projection $\hat{n}_{i,1} - \hat{n}_{i,2} = 0$ automatically implies a SU(2) gauge constraint:

$$c^\dagger_{i,\alpha}\tau^l_{\alpha,\beta}c_{i,\beta} = 0, \quad l = 1, 2, 3. \tag{16}$$

The reason is that, in the Fock space of $c_{i,1}$ and $c_{i,2}$, the only states that survive the projection are $|0,0\rangle_i$, and $c^\dagger_{i,1}c^\dagger_{i,2}|0,0\rangle_i$, both are singlets under a local SU(2) rotation on $[c_{i,1}; c_{i,2}]$. This SU(2) gauge structure emerges *only* when $N = 1$, i.e., when the original state $|\Psi_0\rangle$ is a spinless fermion state.

Motivated by experiments in candidate spin liquid materials, one of the spin liquid states discussed most is the "spinon Fermi surface state" [7,51–60]. A "spinon Fermi surface" state can be naturally produced in our context when the original state $|\Psi_0\rangle$ is a spinless fermion state with a Fermi surface. Note that in our current case, the Fermi surface state after projection would most naturally have a SU(2) gauge symmetry as long as H_0 is real, rather than a U(1) gauge symmetry as was often discussed in the literature of spin liquids.

A $(2+1)d$ Fermi surface coupled to a dynamical bosonic field (e.g., a dynamical gauge field) is a challenging theoretical problem in general, and it has continuously attracted enormous theoretical interest and efforts [61–72]. In our current case, intuitively, the SU(2) gauge field would yield an attractive interaction between c_1 and c_2, and this attractive interaction may lead to pairing instability, i.e., in the doubled Hilbert space there could be condensate of the Cooper pair operator $\hat{\Delta} = c_1c_2$. A Cooper pair condensate leads to the following long-range correlation:

$$\langle\langle\rho_\infty|(c_1c_2)_0(c_1c_2)^\dagger_{\boldsymbol{x}}|\rho_\infty\rangle\rangle = \mathrm{tr}\Big(c^\dagger_0\rho_\infty c_0 c^\dagger_{\boldsymbol{x}}\rho_\infty c_{\boldsymbol{x}}\Big) \xrightarrow{x\to\infty} \mathrm{const.} \tag{17}$$

Superconductivity of matter fields coupled to a non-Abelian gauge field is referred to as color superconductivity (for reviews, see Refs. [73,74]). The superconductivity can be obtained through an ϵ expansion in theory [75,76], and an extrapolation to $\epsilon = 1$. An experimental realization of the state in the quantum simulator can serve as a test for this theoretical prediction.

We have also numerically studied the projected Fermi surface state with two flavors of fermions, and indeed we observe a long range correlation of Cooper pairs (Fig. 2), consistent with our theoretical prediction.

In the doubled space, a Cooper pair condensate spontaneously breaks the charge $\mathrm{U}(1)_e$ symmetry, which is also the so-called "strong" U(1) symmetry of the density matrix [77,78]. The operator $c^\dagger_{\boldsymbol{x}}\rho c_{\boldsymbol{x}}$ transforms nontrivially under the strong U(1) symmetry, but invariant under the "weak" U(1) symmetry. Hence, the long-range correlation of Eq. (2) implies a "strong-to-weak" spontaneous breaking (SW SSB) of the U(1) symmetry in the mixed state, a subject that has attracted enormous interest recently [16–20]. The SW SSB physically can be viewed as a

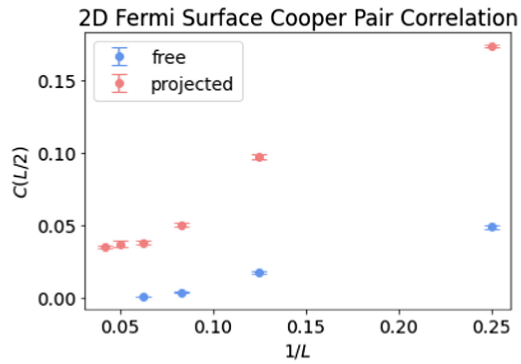

FIG. 2. The projected free fermion wave function with two flavors of fermions and a Fermi surface. There is a long range correlation of Cooper pair, as expected. The numerics is performed on a square lattice tight binding model with nearest neighbor and second neighbor hopping, $t_2/t_1 = 0.2$. We fix the system size at $L \times L$, and measure Cooper pair correlation at $r = L/2$.

signature of a quantum system becoming more classical, through interacting with the environment, or measurement. In the Keldysh formalism, the SW SSB corresponds to the "locking" of the forward and backward paths through condensation of a field that couples the two paths; the condensed field precisely corresponds to the superconductor order parameter in our case.

Summary and discussion—In this Letter, we demonstrated that starting with a simple fermion wave function, the steady state density matrix of the Lindbladian evolution, or the ensemble generated from strong measurement of local densities, is a Gutzwiller projected wave function in the doubled Fock space. The Gutzwiller projection has been broadly used as a numerical trial wave function of spin liquid. We propose that this new construction of spin liquids can be realized in real experimental platforms, e.g., the Fermi gas microscope. We also predict that in certain scenarios, the constructed mixed state has a strong-to-weak spontaneous U(1) symmetry breaking, which corresponds to a superconductor in the doubled Fock space. Predictions made by our understanding of spin liquids, as well as the detailed measurement protocol of our predictions, are described in SM [21].

With Hermitian jump operators, the maximally mixed density matrix is always one of the steady states. For local dephasing that we considered, local density configurations are always conserved. This is why the identity matrix is not the only solution of the steady state, and the true steady state will depend on the initial state. At time t, the deviation of the particle number on a given site from the Gutzwiller projection is approximately $\exp(-\gamma t)$ [Eq. (4)]. Hence, we need $t \gg 1/\gamma$ for the system to be well approximated by the Gutzwiller wave function. If we turn on perturbations with strength g in the Lindbladian that do not commute with local density, their effect will become significant for $t > 1/g$. Hence, as long as $g \ll \gamma$, there is a window at finite time where we can observe the desired physics of spin liquid.

We have focused on mixed states prepared from initial *gapless* states $|\Psi_0\rangle$. Gapped fermion states can also be very interesting—even free fermion insulators can feature nontrivial topology, e.g., the Chern insulator. The Choi representation of the Chern insulator in the doubled space is a quantum spin Hall state, and the procedure proposed in this work will lead to a Gutzwiller projection of the QSH state. We leave the generalizations of our work to topological insulators to a future work.

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